

## HIGHER-ORDER QCD CORRECTIONS TO DEEP-INELASTIC SUM RULES

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### ABSTRACT

The brief review of the current status of the studies of the effects of the higher-order perturbative QCD corrections to the deep-inelastic sum rules is presented.

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## 1. Introduction

Up to recently the consideration of the QCD predictions for the structure functions (SFs) of deep-inelastic scattering (DIS) was the basic source of information about the structure of a nucleon. However, the recent measurements of the SFs of both polarized and non-polarized DIS [1],[2] in the wide interval of the  $x = Q^2/2pq$  variable open the possibility of a more precise determination of the number of the DIS sum rules (SRs), namely of the polarized Bjorken SR  $BjpSR = \int_0^1 g_1^{ep-en}(x, Q^2)dx$ , the polarized Ellis-Jaffe SR  $EJSR = \int_0^1 g_1^{p(n)}(x, Q^2)dx$  and of the non-polarized Gross-Llewellyn Smith SR  $GLSSR = (1/2) \int_0^1 F_3^{\nu p+\bar{\nu}p}(x, Q^2)dx$ . In view of this experimental progress the detailed studies of the theoretical predictions for the DIS SRs started to attract special attention. In this talk we concentrate on the discussions of the effects of the perturbative QCD corrections to these quantities.

## 2. The polarized Bjorken SR

The theoretical expression for the  $BjpSR$  has the following form

$$BjpSR = \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - a \left( 1 + \sum_{i \geq 1} d_i a^i \right) + O\left(\frac{1}{Q^2}\right) \right], \quad (1)$$

where  $a = \alpha_s/\pi$  and the exact expressions for the coefficients  $d_1$  and  $d_2$ , namely  $d_1^{ex} = 4.583 - 0.333f$  and  $d_2^{ex} = 41.440 - 7.607f + 0.177f^2$ , were calculated in the  $\overline{MS}$  scheme in Refs. [3] and [4] respectively. However, this scheme is not the unique prescription for fixing the renormalization scheme ambiguities. For example, one can use the principle of minimal sensitivity (PMS) [5] or the effective charges (ECH) approach [6]. These methods assume the role of “optimal” prescriptions, in the sense that they might provide better convergence of the corresponding approximations for physical quantities in the non-asymptotic regime. Therefore, applying these methods, one can try to estimate the effects of the  $O(a^{N+1})$  corrections starting from the  $N$ -th order approximation  $D_N^{opt}(a_{opt})$ . As was originally explained in Ref. [5], rewriting the  $N$ -th order optimized expression  $D_N^{opt}(a_{opt})$  of the physical quantity in terms of the coupling constant  $a$  of the initial scheme one can get the following relation  $D_N^{opt}(a_{opt}) = D_N(a) + \delta D_N^{opt} a^{N+1}$ . It is now possible to consider the term  $\delta D_N^{opt}$  as the one, that simulates the coefficient of the  $O(a^{N+1})$  correction to the physical quantity  $D(a)$  calculated in the certain initial scheme. Its concrete form  $\delta D_N^{opt} = \Omega_N(d_i, c_i) - \Omega_N^{opt}(d_i^{opt}, c_i^{opt})$  ( $1 \leq i \leq N-1$ ) depends on the coefficients  $d_i$  of the physical quantity and  $c_i$  of the QCD  $\beta$ -function defined as  $\beta(a) = -\beta_0 a^2 (1 + \sum_{i \geq 1} c_i a^i)$ . The correction terms  $\Omega_N^{opt}$  reflect the dependence on the way of realization of the “optimal” prescription and are rather small. Within the ECH approach one has  $\Omega_N^{opt} = 0$ . Moreover, in the case of the PMS approach,  $\Omega_3^{PMS} = 0$  [7] and therefore  $\delta D_3^{ECH} = \delta D_3^{PMS}$ .

The above-mentioned procedure was recently used to estimate the  $O(a^4)$  correction to the  $BjpSR$  [8, 9] and to roughly fix the uncertainty in the value of the  $O(a^5)$  term [9]. The table, taken from Ref. [9], summarizes the results of estimates of the coefficients  $d_i^{est}$ , obtained by re-expansion of the ECH for the  $BjpSR$  into the initial  $\overline{MS}$  scheme, and demonstrates their dependence on the number of flavours  $f$ . We consider the satisfactory agreement of the obtained estimates  $d_2^{est}$  with the results  $d_2^{ex}$  of Ref. [4] as an argument in favour of the applicability of this procedure.

$f$	$d_2^{ex}$	$d_2^{est}$	$d_3^{est}$	$d_4^{est} - c_3 d_1$
1	34.01	27.25	290	2557
2	26.93	23.11	203	1572
3	20.21	19.22	130	854
4	13.84	15.57	68	342
5	7.83	12.19	18	27
6	2.17	9.08	-22	-135

The results of estimates of the  $NNLO$ ,  $N^3LO$  and  $N^4LO$  corrections in the series for  $BjpSR$ .

The existing ambiguities in  $d_4^{est}$  are related to the assumption used that the real value of  $d_3$  does not differ from  $d_3^{est}$  and to the lack of knowledge of the 4-loop coefficient of the QCD  $\beta$ -function. However, even without application of any additional assumption about its value (e.g. for  $f = 3, 4, 5$  one can use the “geometric progression” guess  $c_3 = c_2^2/c_1$ ), it is possible to conclude that in the currently available region of energies  $Q^2 = 2-10 \text{ GeV}^2$  the higher-order QCD corrections to the  $BjpSR$  are not negligibly small. The results discussed were used in the process of the determination of the value of  $\alpha_s(M_Z) = 0.122_{-0.009}^{+0.005}$  [10] using the  $BjpSR$  measurements [1]. This result should be compared with the result  $\alpha_s(M_Z) = 0.115 \pm 0.005(exp) \pm 0.003(th)$  extracted in Ref. [11] from the  $GLSSR$  data [2]. Its theoretical uncertainty comes from the uncertainty of the estimates [12] of the higher-twist terms.

### 3. The Ellis-Jaffe SR

The theoretical expression for the  $EJSR$  consists of two parts:  $EJSR(Q^2) = EJ_{NS}(Q^2) + EJ_{SI}(Q^2)$ . The first non-singlet part is a renormalization-group-invariant quantity and, apart from the overall factor, coincides with  $BjpSR$ . For the case of  $f = 3$  active flavours, one has [8, 9]:

$$EJ_{NS}^{p(n)} = \left[ 1 - a - 3.583a^2 - 20.215a^3 - 130a^4 - O(a^5) \right] \times (\pm a_3/12 + a_8/36) + O\left(\frac{1}{Q^2}\right), \quad (2)$$

where  $a_3 = \Delta u - \Delta d$ ,  $a_8 = \Delta u + \Delta d - 2\Delta s$  and  $\Delta u$ ,  $\Delta d$ ,  $\Delta s$  can be interpreted as the measure of the polarization of the quarks in a nucleon. The  $O(a^2)$  correction to  $EJ_{SI}$ , which contains the anomalous-dimension term, was calculated recently [13]. In order to estimate the value of the  $O(a^3)$  correction the methods of Ref. [5] were supplemented by the considerations of the quantities with anomalous dimensions [14]. For  $f = 3$  numbers of flavours, the estimates were obtained [15] for the renormalization-invariant definition of the singlet contribution and in the case when the  $Q^2$ -dependence of  $\Delta\Sigma = \Delta u + \Delta d + \Delta s$  is specified. The more definite renormalization-invariant estimates have the following form [15]:

$$EJ_{SI} = \left[ 1 - 0.333a - 0.549a^2 - 2a^3 \right] \frac{1}{9} \Delta\Sigma_{inv} + O\left(\frac{1}{Q^2}\right). \quad (3)$$

In order to obtain the  $O(\alpha_s^3)$  estimates in the case when the  $Q^2$ -dependence of  $\Delta\Sigma = \Delta u + \Delta d + \Delta s$  is specified, it is necessary to fix the value of the 4-loop coefficient of the corresponding anomalous dimension function, which starts its expansion from the  $O(a^2)$  level, namely  $\gamma(a) =$

$\sum_{i \geq 1} \gamma_i a^{i+1}$  [15]. For the case of  $f = 3$  the final result [15] reads:

$$EJ_{SI} = \left[ 1 - a - 1.096a^2 - 3.7a^3 \right] \frac{1}{9} \Delta \Sigma(Q^2) + O\left(\frac{1}{Q^2}\right). \quad (4)$$

Note that we used an additional assumption about the value of the non-calculated term  $\gamma_3$  [15]:  $\gamma_3 \approx \gamma_2^2/\gamma_1$ , whereas the expression for the  $\gamma_2$ -term is known [16].

It can be seen that the perturbative contributions to Eqs. (3),(4) are significantly smaller than the coefficients of Eq. (2). This fact has an important phenomenological consequence, namely the possibility of describing available data of Ref.[1] by allowing  $\Delta s$  to be non-zero [10]. The outcomes of the fits [10] are:  $\Delta s = -0.10 \pm 0.03$ ,  $\Delta \Sigma = 0.31 \pm 0.07$  at  $Q^2 = 10 \text{ GeV}^2$ . Note, however, that in view of the controversial claims about the possible contributions of the higher-twist terms in Eqs. (2)-(4) [17], the analysis of the polarized DIS data [1] deserves further experimental and theoretical studies. One of the possible clarifying advancements could be the determination, from the experimental data, of the  $Q^2$ -dependence of the  $BjpSR$  and of the  $EJSR$ . In the case of the  $GLSSR$  this work was already started [18]. Another important theoretical problem is related to the study of the consequences of the manifestation of the axial anomaly in the theoretical expression for the  $EJSR$  [19].

#### 4. DIS vs. $e^+e^-$ annihilation

The new non-trivial connection between the characteristics of the  $e^+e^-$  annihilation and DIS was discovered recently [20]:

$$D^{e^+e^-}(Q^2) \times GLSSR(Q^2) \sim 1 + \frac{\beta(a)}{a} \left[ C_1 a + C_2 a^2 \right] + O(a^4) \quad (5)$$

These characteristics are the analytical  $O(a^3)$  approximations of the function  $D^{e^+e^-}(Q^2)$  [21, 22] and of the  $GLSSR$  [4]. In Eq.(5)  $C_1$  and  $C_2$  are the analytical numbers, which depend on the structure of the non-Abelian gauge group. It should be stressed that the corresponding perturbative expression for the  $GLSSR$  equals the one of the  $BjpSR$  plus the  $O(a^3)$  contribution of the light-by-light-type graphs [4]. Equation (5) demonstrates the appearance of the radiative corrections in quark-parton formula of Ref. [23] starting from the  $O(a^2)$  level. This formula is connecting the amplitude, related to the axial anomaly, for the  $\pi^0 \rightarrow \gamma\gamma$  decay with the quark-parton expressions for the  $D^{e^+e^-}$  function and the  $BjpSR$ . The most interesting, yet non-explained, feature of Eq.(5) is the factorization of the factor  $\beta(a)/a$  at the  $O(a^3)$ -level. We believe that the detailed study of this relation could have important theoretical and phenomenological consequences.

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